NAG Toolbox for MATLAB

f08ba

1 Purpose

f08ba computes the minimum norm solution to a real linear least-squares problem

$$\min_x \|b - Ax\|_2$$

using a complete orthogonal factorization of A. A is an m by n matrix which may be rank-deficient. Several right-hand side vectors b and solution vectors x can be handled in a single call.

2 Syntax

```
[a, b, jpvt, rank, info] = f08ba(a, b, jpvt, rcond, 'm', m, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

The right-hand side vectors are stored as the columns of the m by r matrix B and the solution vectors in the n by r matrix X.

f08ba first computes a QR factorization with column pivoting

$$AP = Q\begin{pmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{pmatrix},$$

with R_{11} defined as the largest leading sub-matrix whose estimated condition number is less than $1/\mathbf{rcond}$. The order of R_{11}, **rank**, is the effective rank of A.

Then, R_{22} is considered to be negligible, and R_{12} is annihilated by orthogonal transformations from the right, arriving at the complete orthogonal factorization

$$AP = Q\begin{pmatrix} T_{11} & 0 \\ 0 & 0 \end{pmatrix}Z.$$

The minimum norm solution is then

$$X = PZ^{\mathrm{T}}\begin{pmatrix} T_{11}^{-1}Q_1^{\mathrm{T}}b \\ 0 \end{pmatrix}$$

where Q_1 consists of the first **rank** columns of Q.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

2: **b**(**ldb**,$*$) – **double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The m by r right-hand side matrix B.

3: **jpvt**($*$) – **int32 array**

Note: the dimension of the array **jpvt** must be at least $\max(1, \mathbf{n})$.

If $\mathbf{jpvt}(i) \neq 0$, the ith column of A is permuted to the front of AP, otherwise column i is a free column.

4: **rcond** – **double scalar**

Used to determine the effective rank of A, which is defined as the order of the largest leading triangular sub-matrix R_{11} in the QR factorization of A, whose estimated condition number is $< 1/\mathbf{rcond}$.

5.2 Optional Input Parameters

1: **m** – **int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – **int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **nrhs_p** – **int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrices B and X.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, work, lwork

5.4 Output Parameters

1: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

a has been overwritten by details of its complete orthogonal factorization.

2: **b**(**ldb**,$*$) – **double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{m}, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

the n by r solution matrix X.

3: **jpvt**$(*)$ – **int32 array**

Note: the dimension of the array **jpvt** must be at least $\max(1, \mathbf{n})$.

If $\mathbf{jpvt}(i) = k$, then the ith column of AP was the kth column of A.

4: **rank** – **int32 scalar**

The effective rank of A, i.e., the order of the sub-matrix R_{11}. This is the same as the order of the sub-matrix T_{11} in the complete orthogonal factorization of A.

5: **info** – **int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **jpvt**, 9: **rcond**, 10: **rank**, 11: **work**, 12: **lwork**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

See Section 4.5 of Anderson *et al.* 1999 for details of error bounds.

8 Further Comments

The complex analogue of this function is f08bn.

9 Example

```
a = [-0.09, 0.14, -0.46, 0.68, 1.29;
     -1.56, 0.2, 0.29, 1.09, 0.51;
     -1.48, -0.43, 0.89, -0.71, -0.96;
     -1.09, 0.84, 0.77, 2.11, -1.27;
     0.08, 0.55, -1.13, 0.14, 1.74;
     -1.59, -0.72, 1.06, 1.24, 0.34];
b = [7.4;
     4.2;
     -8.300000000000001;
     1.8;
     8.6;
     2.1];
jpvt = [int32(0);
     int32(0);
     int32(0);
     int32(0);
     int32(0)];
rcond = 0.01;
[aOut, bOut, jpvtOut, rank, info] = f08ba(a, b, jpvt, rcond)
```

```
aOut =
   -2.9366   -0.7273    1.9134   -1.0317   -0.0890
    0.5234    2.8681    0.1721    0.7329   -0.1692
    0.4966   -0.4778   -2.3028   -0.6896    0.1052
    0.3657   -0.5520   -0.3306    1.2344   -0.3173
   -0.0268    0.6259   -0.0197    0.1934   -0.0034
    0.5335   -0.0259    0.0087   -0.3961   -0.6352
bOut =
    0.6344
    0.9699
   -1.4402
    3.3678
    3.3992
   -0.0035
jpvtOut =
           1
           5
           4
           2
           3
rank =
           4
info =
           0
```